Concerning the Landau pole in 3–3–1 models

A.G. Dias[1,a], R. Martinez[2,b], V. Pleitez[3,c]

[1] Instituto de Física, Universidade de São Paulo, C.P. 66.318, 05315-970 São Paulo, SP, Brazil
[2] Departamento de Física, Universidad Nacional de Colombia, Bogotá, Colombia
[3] Instituto de Física, Teórica, Universidade Estadual Paulista, Rua Pamplona 145, São Paulo, SP, Brazil

Abstract. Some 3–3–1 models predict the existence of a non-perturbative regime at the TeV scale. We study in these models and their supersymmetric extensions, the energy at which the non-perturbative limit and a Landau-like pole arise. An order of magnitude for the mass of the extra neutral vector boson, Z', present in these models is also obtained.

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1 Introduction

The so called 3–3–1 extensions of the standard model (SM) are interesting options for the physics at the TeV scale [1,2]. Although these models coincide at low energies with the SM, they explain some fundamental questions that are accommodated, but not explained by the former. For instance, i) in order to cancel chiral anomalies the number of generation N_g must be a multiple of three, but because of the asymptotic freedom in QCD, which implies that the number of generations must be $N_g \leq 5$, it follows that in those models the only number of generations allowed is $N_g = 3$; ii) these models as any with $SU(3)_W$ symmetry explain why $\sin^2 \theta_W < 1/4$ at the Z-pole (see below); iii) the electric charge is quantized independently of the nature of neutrinos [3]; iv) the Peccei-Quinn symmetry is almost an automatic symmetry of the classical Lagrangian [4] and with a minimal modification, the PQ symmetry as the solution to the strong CP problem is automatically implemented and the axion is protected against semiclassical gravity effects [5]; v) the theory becomes non-perturbative at the TeV scale, and the same happens with the respective $\mathcal{N} = 1$ supersymmetric version [6]. There are other models with $SU(3)_W$ symmetry [7], but some of them imply charged heavy bileptons which are stable, that in turn lead to potentially cosmological troubles [8]. Another interesting possibility to consider is by introducing extra dimensions [9] and the orbifold compactification [10,11], or other sort of 3–3–1 models as in [12]. A common feature of models with $SU(3)_W$ electroweak symmetry is the existence of simply and doubly charged or neutral non-hermitian vector bosons. For instance, the doubly charged and neutral

ᵃ e-mail: alexdias@fma.if.usp.br
ᵇ e-mail: remartinezm@unal.edu.co
ᶜ e-mail: vicente@ift.unesp.br

vector bosons can be discovered by measurement of the left-right asymmetries in lepton-lepton scattering [13,14], in muonium-antimuonium transitions [15,16] or in accelerator processes [17,18].

One of the main feature of these models is the fact that when the g_L and g_X coupling constants of the gauge groups $SU(3)_L$ and $U(1)_X$, respectively, are related with the electroweak mixing angle, the following relation is obtained

$$\frac{g_X^2}{g_L^2} = \frac{\sin^2 \theta_W}{1 - 4\sin^2 \theta_W} \tag{1}$$

in the models of [1,2]. When $\sin^2 \theta_W(\mu) = 1/4$ the coupling constant $g_X(\mu)$ becomes infinite, i.e., a Landau-like pole arises, however the theory loses its perturbative character even at an energy scale lower than μ. The other possibility $g_L \to 0$ is ruled out since g_L is the same as in the standard model, g_2, due to the fact that the $SU(2)_L$ subgroup is totally embedded into $SU(3)_L$.

The possible existence of a Landau-like pole in 3–3–1 models is not unexpected since every non-assintotically free theory seems to has such a behavior. The new feature is that in some of these models that behavior may happens at energies of just few TeVs. This will imply that the cutoff, $\Lambda_{\mathrm{cutoff}}$, in the theory can not be eliminated, by taking $\Lambda_{\mathrm{cutoff}} \to \infty$, as it is expected in renormalizable theories. In this limit the theory might be a trivial theory. This is supposed to be the case of pure QED since the works of Landau and co-workers [19]. From the phenomenological point of view this result is not very dangerous, we already know that QED has to be embedded in the electroweak theory at a few hundred GeVs, and also that weak and strong correction have to be taken into account in the calculations of physical observables, even those that are purely electromagnetic in origin, like the $(g-2)_\mu$ factor, etc.

However, as a mathematical laboratory it is interesting to study pure QED at arbitrary small distances. In fact, lattice calculation suggests that chiral symmetry breaking allows QED to escape the Landau pole problem [20, 21]. This should happen because the chiral symmetry breaking is always strong enough to push the Landau pole above the cutoff [20]. It is interesting that the possibility of the existence of the Landau pole, or that the theory is a trivial one, arises already at the lower order in perturbation theory. More sophisticated calculations only enhance our confidence that this phenomenon is not an effect of the perturbation theory at lowest order. This is in accord with the point of view that the renormalization group provide qualitative guidance with respect to the asymptotic behavior at very high energies even where coupling constant at the scale of interest are too large to allow the use of perturbation theory. In particular this method provides usual insight into the types of possible behavior in field theories [22].

Notwithstanding, we must remember that both QED and the standard model are effective, and not fundamental theories. It means that effective operators with dimensions higher that $d = 4$ have to be considered if we want, for instance, to get a realistic continuum limit in lattice calculations [22]. Thus it seem that using the pure versions of these models are still inconclusive and the renormalization group may give an insight in this issue in 3–3–1 models.

The possibility of triviality implies that new phenomena must enter before the reach the Landau pole: or a new phase of the theory or the theory must be embedded in a more general one. This has been used in the context of the standard model to constraint the upper value of the Higgs boson mass [23]. Moreover, recently it has been argued that this upper limit on the Higgs bosons mass does not come from an instability of the vacuum [24].

In the past years some authors, using perturbation theory, have calculated the energy scale at which the weak mixing angle get the 0.25 value [25–27]. They have been found, taking into account only the degrees of freedom of the standard model, that this condition occurs at an energy scale of the order of 3–4 TeV in the model of [1]. This value is an upper limit of the energy scale at which the Landau-like pole occurs (see the discussion in Sect. 6).

Our goal in this paper is to study the running of $\sin^2 \theta_W$ with energy in 3–3–1 models [1,2] and their supersymmetric extensions given in [6]. However, since we have verified that, with the representation content of the minimal models g_L does not change significantly, we study the running of g_X. We confirm the order of magnitude of the results of the previous works but we considered a more general scenario, when the $SU(3)_L$ symmetry breakdown scale, μ_{331}, is inside the perturbative range and when the exotic quarks or SUSY particles are considered much heavier than the other particles. As previous calculations, ours are also done at the 1-loop level but we briefly comment the 2-loop case. For this reason our result have to be seen as an estimative of the energy scale where $\sin^2 \theta_W = 0.25$. Using the perturbation theory to find a singularity could appears self-contradictory, however, we recall that this behavior at relatively low energy, arises because of the constraint in (1). We think that our calculations as the previous ones in 3–3–1 models are only preliminary results. When lattice calculations, or other more appropriate techniques, were available they could be compared with those 1-loop calculations, as is usually done in the $\lambda\phi^4$ or QED cases [24].

The outline of this paper is as follows. In Sect. 2 we revise briefly the minimal representation content of two 3–3–1 models that will be considered next. In Sect. 3 we give the evolution equation and calculate the b_i coefficients in each model with and without supersymmetry. In this section we also reproduce the calculations of [25–27] of the energy at which the condition $s_W^2 = 0.25$ is satisfied, taking into account only the degrees of freedom of the standard model. In Sect. 4, we study the evolution of $\alpha_X \equiv g_X^2/4\pi$, and calculate the energy scale, M', at which $\alpha_X(M') > 1$. We also compare this energy with Λ defined as $\alpha_X(\Lambda) = \infty$, taking into account the degrees of freedom of the 3–3–1 models for energies above an energy scale that we denote μ_{331}. The last section is devoted to our conclusions.

2 3–3–1 Models
with doubly charged vector bosons

The models that we will take into account in this section are characterized by the electric charge operator,

$$\mathcal{Q}_A = \frac{1}{2}\left(\lambda_3 - \sqrt{3}\lambda_8\right) + X, \qquad (2)$$

with two different representation content in the leptonic sector that are giving either by $\Psi_{aL} = (\nu_a, l_a, E_a^+)_L^T \sim (\mathbf{1}, \mathbf{3}, 0)$ (Model A) or by $\Psi_{aL} = (\nu_a, l_a, l_a^c)_L^T \sim (\mathbf{1}, \mathbf{3}, 0)$ (Model B), $a = e, \mu, \tau$. Both models contain doubly charged vector bosons. In the first case, we have to add singlets $l_{aR} \sim (\mathbf{1}, \mathbf{1}, -1)$, $E_{aR} \sim (\mathbf{1}, \mathbf{1}, +1)$; and neutrinos $\nu_{aR} \sim (\mathbf{1}, \mathbf{1}, 0)$, if necessary. In the second case only right-handed neutrinos have to be added, also if necessary. However, since neutral singlet representations do not affect the running of the coupling constants, we will not worry about them here. In both models the quarks transform as follows: $Q_{iL} = (d_i, u_i, j_i)_L^T \sim (\mathbf{3}, \mathbf{3}^*, -1/3)$; $i = 1, 2$; $Q_{3L} = (u_3, d_3, J)_L^T \sim (\mathbf{3}, \mathbf{3}, 2/3)$, with the singlets $u_{\alpha R} \sim (\mathbf{1}, \mathbf{1}, 2/3)$, $d_{\alpha R} \sim (\mathbf{1}, \mathbf{1}, -1/3)$, $\alpha = 1, 2, 3$, $j_{iR} \sim (\mathbf{1}, \mathbf{1}, -4/3)$, and $J_R \sim (\mathbf{1}, \mathbf{1}, 5/3)$,

In these models there are fields with masses of the order of magnitude of the $SU(3)_L$ energy scale. For instance, in Model A the scalar fields necessary to break the gauge symmetry down to $U(1)_Q$ and giving the correct mass to all fermions in the model are three triplets: $\eta = (\eta^0, \eta_1^-, \eta_2^+)^T \sim (\mathbf{1}, \mathbf{3}, 0)$, $\rho = (\rho^+, \rho^0, \rho^{++}) \sim (\mathbf{1}, \mathbf{3}, 1)$ and $\chi = (\chi^-, \chi^{--}, \chi^0) \sim (\mathbf{1}, \mathbf{3}, -1)$. We will denote the vacuum expectation values as follows: $\langle\eta^0\rangle = u/\sqrt{2}$, $\langle\rho^0\rangle = v/\sqrt{2}$ and $\langle\chi^0\rangle = w/\sqrt{2}$. In Model B it is necessary to add an scalar sextet $S \sim (\mathbf{1}, \mathbf{6}, 0)$

$$S = \begin{pmatrix} \sigma_1^0 & h_1^- & h_2^+ \\ h_1^- & H_1^{--} & \sigma_2^0 \\ h_2^+ & \sigma_2^0 & H_2^{++} \end{pmatrix}, \qquad (3)$$

and we will use the notation $\langle \sigma_2^0 \rangle = v_2/\sqrt{2}$. It is also possible to have $\langle \sigma_1^0 \rangle \neq 0$ giving to the neutrinos a Majorana mass. We will not be concerned with this here.

In Model A the physical scalar spectra are such that a singly charged scalar, which is a linear combinations of η_1^+ and ρ^+, has the square mass [28]

$$M_{1+}^2 \propto \left(u^2 + v^2 - \frac{fwv}{u} - \frac{fuw}{v} \right) , \qquad (4)$$

and the other singly charged scalar is a linear combination of η_2^+ and χ^+ and has a square mass

$$M_{2+}^2 \propto \left(w^2 + v^2 - \frac{fvw}{u} - \frac{fuv}{w} \right) , \qquad (5)$$

where $f < 0$ is the trilinear coupling with dimension of mass. We see that even if $f = 0$ only one of the singly charged scalar has a low mass, the other one has a mass square proportional to w^2, so it is heavy enough (unless it is fine tuned) to be decoupled from low energy physics (below the breaking of the $SU(3)_L$ symmetry). The same happens with the doubly charged physical scalar, a linear combination of ρ^{++} and χ^{++}, with a mass square

$$M_{++}^2 \propto \left(w^2 + v^2 - \frac{fuv}{w} - \frac{fuw}{v} \right) , \qquad (6)$$

and we see that it is also too heavy (unless fine tuned) and it will not be considered at low energies. On the other hand, in the real scalar fields sector, there are three physical scalar Higgs, two states that do not depend on w and f having square masses of the order of $u^2 + v^2$ and the other one is heavy. It means that at low energies we consider two scalar doublets of $SU(2)_L$ when we use the standard model degrees of freedom. The vector bosons, V^-, U^{--}, the exotic quarks and scalar singlets, all of them may be heavy since their masses are proportional to w. The extra neutral vector boson Z_2, has an even higher mass and also it will not be considered at low energies. This vector bosons is a mixture of Z and Z', but if we neglect this mixture $Z_2 \approx Z'$.

3 The evolution equations

The evolution equations of the coupling constants at the one loop level are given by

$$\frac{1}{\alpha_i(M)} = \frac{1}{\alpha_i(M_Z)} + \frac{1}{2\pi} \, b_i \, \ln\left(\frac{M_Z}{M} \right) , \quad i = 1, 2, 3; \quad (7)$$

where $\alpha_i = g_i^2/4\pi$ and g_3, g_2, g_1 are the coupling constant of the $SU(3)_C, SU(2)_L, U(1)_Y$ gauge groups, respectively. In the context of 3–3–1 models we define the b_i coefficients corresponding to the coupling constants g_3, g_L, g_X of the gauge groups $SU(3)_C, SU(3)_L, U(1)_X$, respectively.

For a general $SU(N)$ gauge group the b_i coefficients are given by

$$b_i = \frac{2}{3} \sum_{\text{fermions}} T_{Ri}(F) + \frac{1}{3} \sum_{\text{scalars}} T_{Ri}(S) - \frac{11}{3} \, C_{2i}(G) \quad (8)$$

for Weyl fermions and complex scalars, and $T_R(I)\delta^{ab} = \mathrm{Tr}[T^a(I)T^b(I)]$ with $I = F, S$; $T_R(I) = 1/2$ for the fundamental representation, $C_2(G) = N$ for $SU(N)$ and $C_2(G) = 0$ for $U(1)$. For $U(1)_y$ we use $\sum T_{R1}(F, S) = \sum y^2$ where $y = Y/2$ for the standard model and $y = X$ for the 3–3–1 models. On the other hand for the respective supersymmetric version we have

$$b_i^{\text{susy}} = \sum_{\text{fermions}} T_{Ri}(F) + \sum_{\text{scalars}} T_{Ri}(S) - 3 \, C_{2i}(G) , \quad (9)$$

and only the usual non-supersymmetric fields are counted now.

We will assume that the standard model with several scalar multiplets is valid until an energy scale μ_{331}, i. e., below μ_{331} we consider the SM plus some light scalar doublets or triplets. A $SU(2)_L \otimes U(1)_Y$ model with $N_g = 3$ fermion generations, N_H scalar doublets ($Y = \pm 1$) and N_T non-hermitian scalar triplets ($Y = 2$), using (8) and the representation content above, implies

$$b_1 = \frac{1}{6} N_H + N_T + \frac{20}{3} ,$$
$$b_2 = \frac{1}{6} N_H + \frac{2}{3} N_T - \frac{10}{3} ,$$
$$b_3 = -7 . \qquad (10)$$

Notice that we have not used a grand unification normalization for the hypercharge Y assignment. Since we will assume that $\mu_{\text{susy}} \approx \mu_{331}$ when considering the SUSY extensions of a 3–3–1 model, below μ_{331} the only effect of supersymmetry will be the addition of light scalar multiplets. Above μ_{331}, we have to consider the degrees of freedom of the 3–3–1 models.

The heavy leptons, E_a in Model A, quarks J and j_i; the scalar singlets η_2^+, ρ^{++}; the scalar doublets like (χ^-, χ^{--}); and finally, the vector doublets like (V^-, Y^{--}) and the extra neutral vector boson, Z', will not be considered as active degrees of freedom below μ_{331}. Hence, in non-SUSY Model A we have $N_H = 2$ and $N_T = 0$; in the SUSY version $N_H = 4$ and $N_T = 0$. In the non-SUSY Model B we have to take into account the scalar sextet which implies an additional doublet and an non-hermitian triplet, so that $N_H = 3$ and $N_T = 1$, and $N_H = 6$ and $N_T = 2$ in the SUSY case.

In the energy regime below μ_{331} we use the standard definition for $\sin^2 \theta_W(\mu)$:

$$\sin^2 \theta_W(\mu) = \frac{1}{1 + \frac{\alpha_2(\mu)}{\alpha_1(\mu)}} , \quad \mu \leq \mu_{331} . \qquad (11)$$

With this equation for the weak mixing angle and the b_i coefficients in (10), we obtain the value of $\sin^2 \theta_W(\mu_{331})$ using the running equation of $\alpha_{1,2}^{-1}$ given by (7), which will be used as an input for the case of energies above μ_{331}. The values of the energies at which $\sin^2 \hat{\theta}_W(\Lambda) = 0.25$ (at which the curves cut the values 0.25), in models A and B with and without supersymmetry, are shown in Fig. 1. These energies give an order of magnitude of the energy scale of the Landau Pole. In particular, for the non-SUSY
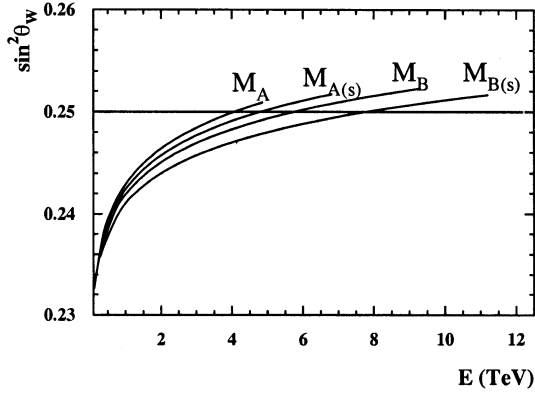
Fig. 1. Running of the electroweak mixing angle for Models A and B considering only the degrees of freedom of the effective 3–2–1 model. (s) stands for the respective supersymmetrized version

Model A (leftmost curve in the figure) we obtain a value for Λ in agreement with that of [25–27], i.e., $\approx 4\,\mathrm{TeV}$.

Next, we consider the case of energies above the scale μ_{331}. Now we have to use the relation

$$\sin^2\theta_W(\mu) = \frac{1}{4}\,\frac{1}{1+\frac{\alpha_L(\mu)}{4\alpha_X(\mu)}} \le 1/4\,, \quad \mu \ge \mu_{331}\,, \quad (12)$$

and the running equation we will be concerned is

$$\frac{1}{\alpha_X(\mu)} = \left[1 - 4\sin^2\hat\theta_W(M_Z)\right]\frac{1}{\alpha(M_Z)} + \frac{1}{2\pi}\,(b_1 - 3b_2)$$

$$\times \ln\left(\frac{M_Z}{\mu_{331}}\right) + \frac{1}{2\pi}b^r_X\,\ln\left(\frac{\mu_{331}}{\mu}\right)\,, \quad (13)$$

where b^r_X, with $r = A, B$, are given below by (14) for Model A, or by (15) for Model B, respectively; with $\sin^2\hat\theta_W(M_Z) = 0.2311$, $\alpha(M_Z) = 1/128$ and $M_Z = 91.188\,\mathrm{GeV}$ [29].

The b_i coefficients in Model A, with and without SUSY, when the degrees of freedom above μ_{331} are taken into account, are given by:

$$b^A_X = 24 + N_\rho + N_\chi\,,$$

$$b^A_{X\slashed{J}} = 10 + N_\rho + N_\chi\,,$$

$$b^{A(\mathrm{susy})}_X = 36 + 3(N_\rho + N_\chi)\,,$$

$$b^{A(\mathrm{susy})}_{X\slashed{J}} = 15 + 3(N_\rho + N_\chi)\,. \quad (14)$$

Similarly, for the case of Model B, we have

$$b^B_X = 20 + N_\rho + N_\chi\,,$$

$$b^B_{X\slashed{J}} = 6 + N_\rho + N_\chi\,,$$

$$b^{B(\mathrm{susy})}_X = 30 + 3(N_\rho + N_\chi)\,,$$

$$b^{B(\mathrm{susy})}_{X\slashed{J}} = 9 + 3(N_\rho + N_\chi)\,. \quad (15)$$

In (14) and (15) we also show the cases when we omit the exotic quarks (this is denoted by $\slashed{J}$ in b_X). We recall that

in the supersymmetric versions we have assumed $\mu_{\mathrm{susy}} \approx \mu_{331}$.

In the minimal non-SUSY version of models A and B, we have $N_\rho = N_\chi = 1$ and $N_\rho = N_\chi = 2$ in the respective SUSY models. Notice that in both models adding more triplet of scalars like ρ and χ enhance b_X and produce a lower value for Λ. In the SUSY version the running is always faster.

4 The Landau pole

We see from (12) that $\alpha_X(\mu) \to \infty$ when $\mu \to \Lambda$, and we have $\sin^2\theta_W(\mu) \to 0.25$. This is the Landau pole that we have mentioned in Sect. 1. In practice we study, using (13), what is the energy $\mu = M'$ at which $\alpha_X(M') > 1$, i.e., the condition when α_X becomes non-perturbative and we compare this result with the energy Λ calculated directly by the expression

$$\Lambda = \mu_{331}\exp\left(\frac{2\pi}{b_X\alpha_X(\mu_{331})}\right)\,, \quad (16)$$

which must coincide with, or be of the same order, the value obtained by using the condition $\sin^2\theta_W(\Lambda) = 0.25$. Of course, we expect that $M' \lesssim \Lambda$.

4.1 The Landau pole in model A

As we said before, at energies below μ_{331} there is an approximated $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ symmetry with the particle content of the SM plus a second scalar doublet, i. e., in the context of Model A: in the fermion sector there are the usual doublets $(\nu_a, l_a)_L \sim (\mathbf{1}, \mathbf{2}, -1)$ and $(u_\alpha, d_\alpha)_L \sim (\mathbf{3}, \mathbf{2}, 1/3)$; the singlets $l_{aR} \sim (\mathbf{1}, \mathbf{1}, -2)$ (and right-handed neutrinos but they do not affect the running of the constants); and $u_{\alpha R} \sim (\mathbf{3}, \mathbf{1}, 4/3)$, $d_{\alpha R} \sim (\mathbf{3}, \mathbf{1}, -2/3)$. In the scalar sector we have two doublets $(\eta^0, \eta^-_1) \sim (\mathbf{1}, \mathbf{2}, -1)$ and $(\rho^+, \rho^0) \sim (\mathbf{1}, \mathbf{2}, +1)$.

These are the degrees of freedom that are active at energies $\mu \le \mu_{331}$. Hence, we have $N_H = 2, N_T = 0$ in the non-SUSY model and $N_H = 4, N_T = 0$ in the SUSY one. With this particle content we have from (10) [for completeness we include the coefficient b_3]:

$$(b_1, b_2, b_3) = (7, -3, -7)\,,$$

$$(b_1, b_2, b_3)^{\mathrm{susy}} = (22/3, -8/3, -7)\,, \quad (17)$$

and using (11) for obtaining $\alpha_X(\mu_{331})$. If we use only the degrees of freedom that were used in obtaining the coefficients above we get, using (11), that $\sin^2\hat\theta_W(\Lambda) = 0.25$ when $\Lambda \approx 4.10\,\mathrm{TeV}$ in the non-SUSY case and $\Lambda \approx 4.8\,\mathrm{TeV}$ in the SUSY case, as can be seen from Fig. 1.

Above the μ_{331} scale, the full representation of the 3–3–1 model have to be taken into account and we get, according to (14) [for future use we have included the coefficients b_L, b_3],

$$(b_X, b_L, b_3)^A = (26, -13/2, -5)\,,$$

Table 1. Values of M' and Λ for the non-SUSY Model A. The number inside parenthesis are the values for the case when we omit the exotic quarks in the running equation. We show, in the last column, an estimative for the mass of the Z' vector boson. All masses are in TeV

μ_{331}	$\hat{\alpha}_X(\mu_{331})$	M'	Λ	$M_{Z'}(\mu_{331})$
2.0	0.55	2.4(3.0)	3.1(5.2)	5.2
1.5	0.39	2.2(3.4)	2.8(5.7)	3.3
1.0	0.28	1.9(3.9)	2.4(6.5)	1.9
0.75	0.23	1.7(4.2)	2.1(7.1)	1.3
0.5	0.19	1.4(4.8)	1.8 (8.2)	0.8

Table 2. Same as Table 1 but for the SUSY Model A

μ_{331}	$\hat{\alpha}_X(\mu_{331})$	M'	Λ	$M_{Z'}(\mu_{331})$
2.0	0.47	2.3(2.6)	2.6(3.3)	4.9
1.5	0.35	1.9(2.3)	2.1(2.9)	3.1
1.0	0.26	1.4(1.9)	1.6(2.4)	1.8
0.75	0.22	1.2(1.7)	1.3(2.1)	1.2
0.50	0.18	0.9(1.4)	1.0 (1.8)	0.75

$$(b_X, b_L, b_3)^A_J = (12, -13/2, -7)\,;$$
$$(b_X, b_L, b_3)^{A(\text{susy})} = (48, 0, 0)\,,$$
$$(b_X, b_L, b_3)^{A(\text{susy})}_J = (27, 0, -3)\,. \tag{18}$$

With the exotic quarks we will consider two situations. First, that they have masses below Λ and are taken into account in the evolution equations in the interval $[\mu_{331}, \Lambda]$; second, we assume that their masses are higher than Λ and are not considered in the running coupling constants.

The result for the non-supersymmetric model appear in Table 1 for different values for the μ_{331} scale: 2.0, 1.5, 1.0, 0.75 and 0.50 TeV. The same is done in Table 2 for the supersymmetric model. In both cases the M' and Λ values when the exotic quarks are considered heavy until the Landau pole scale are shown in parenthesis in the respective table. In the last column we show an order of magnitude of the Z' neutral vector boson (see below).

4.2 The Landau pole in model B

In this case below μ_{331} in the scalar sector we have to consider three doublets $(\eta^0, \eta_1^-) \sim (\mathbf{1}, \mathbf{2}, -1)$, (ρ^+, ρ^0), $(h_2^+, \sigma_2^0) \sim (\mathbf{1}, \mathbf{2}, +1)$; and one non-hermitian triplet $T \sim (\mathbf{1}, \mathbf{3}, +2)$. With this particle content we have from (10), below μ_{331} i.e., we have $N_H = 3$ and $N_T = 1$ ($N_H = 6$, $N_T = 2$ in the SUSY case):

$$(b_1, b_2, b_3) = (49/6, -13/6, -7)\,,$$
$$(b_1, b_2, b_3)^{\text{susy}} = (32/3, -1, -7)\,. \tag{19}$$

If we use only the degrees of freedom that were used in obtaining the coefficients above we get again, from (11), that $\sin^2 \hat{\theta}_W(\Lambda) = 0.25$ when $\Lambda \approx 5.7$ TeV in the non-SUSY

Table 3. Same as Table 1 but for the Model B

μ_{331}	$\hat{\alpha}_X(\mu_{331})$	M'	Λ	$M_{Z'}(\mu_{331})$
2.0	0.40	3.0(6.3)	4.0(13.9)	4.5
1.5	0.32	2.7(8.0)	3.8(17.7)	3.0
1.0	0.24	2.4(11.3)	3.2(24.8)	1.7
0.75	0.21	2.2(14.4)	2.9(31.5)	1.2
0.5	0.17	1.9(20.1)	2.5 (44.2)	0.7

Table 4. Same as Table 2, but for the SUSY Model B

μ_{331}	$\hat{\alpha}_X(\mu_{331})$	M'	Λ	$M_{Z'}(\mu_{331})$
2.0	0.34	2.7(3.5)	3.1(4.8)	4.1
1.5	0.28	2.2(3.2)	2.5(4.4)	2.8
1.0	0.22	1.7(2.8)	1.9(3.8)	1.7
0.75	0.19	1.4(2.5)	1.6(3.4)	1.1
0.5	0.17	1.0(2.2)	1.2(3.0)	0.7

case and $\Lambda \approx 7.8$ TeV in the SUSY case, as can be seen from Fig. 1. If we consider the doublet (h_2^+, σ_2^0) heavy (but keeping its VEV small) we get $b_1 = 8$ and $b_1^{\text{susy}} = 28/3$ and the values for M' and Λ are a little bit smaller than the case considered here.

Above the μ_{331} scale, the full representation of the 3–3–1 model have to be taken into account and we obtain, according to (15) [again for future use we have included again the coefficients b_L, b_3],

$$(b_X, b_L, b_3)^B = (22, -17/3, -5)\,,$$
$$(b_X, b_L, b_3)^B_J = (8, -17/3, -7)\,,$$
$$(b_X, b_L, b_3)^{B(\text{susy})} = (42, 5, 0)\,,$$
$$(b_X, b_L, b_3)^{B(\text{susy})}_J = (21, 5, -3)\,. \tag{20}$$

The results are shown in Table 3 for the non-SUSY Model B and in Table 4 for the respective SUSY model.

5 Z' mass and 2-loop evolution equations

The Z' is the heaviest vector boson of the models, the Landau pole energy scale (Λ) is supposed to be an upper limit for its mass in the context of a perturbative approach [25]. However, the mass of this boson, at the scale μ_{331}, and assuming $\langle \chi^0 \rangle \approx \mu_{331}$ has an order of magnitude given by

$$M_{Z'}(\mu_{331}) \simeq [4\pi \, \alpha_X(\mu_{331})]^{1/2} \, \mu_{331}\,. \tag{21}$$

The values for the estimative of $M_{Z'}$ using (21) are shown in the last column of Tables 1–4. We see that for some values of α_X, $M_{Z'}$ is larger than M' or Λ.

It is interesting to note that in the SUSY version of Model A at energies above μ_{331} the dependence with the energy in $SU(3)_L$ and $SU(3)_C$ is lost since, as can be seen from (18), at the 1-loop we have that $b_L = b_3 = 0$, i.e.,

$$\alpha_L(\mu > \mu_{331}) = \alpha_3(\mu > \mu_{331}) = \text{constant}\,, \tag{22}$$

and the same occurs for SUSY Model B for α_3 as shown in (20). We can wonder if this is an artifact of the 1-loop approximation. Thus, let us consider the two-loop evolution equations that are given by

$$\mu \frac{d\,\alpha_i(\mu)}{d\,\mu} = \frac{1}{2\pi}\left[b_i + \frac{1}{4\pi}\sum_{j=1}^{3}b_{ij}\alpha_j(\mu)\right]\alpha_i(\mu)^2, \quad (23)$$

where we have not considered the Yukawa couplings since in any case their dominant contributions seems to be positive. For example, in SUSY Model A [30]

$$b_{ij} = \begin{pmatrix} \frac{784}{3}+12(N_\rho+N_\chi) & 32+16(N_\rho+N_\chi) & 160 \\ 4+2(N_\rho+N_\chi) & 14+\frac{17}{3}(N_\rho+N_\chi) & 48 \\ \frac{62}{3} & 24 & 48 \end{pmatrix}.(24)$$

In fact, in (23), we have $N_\rho = N_\chi = 2$ since we are considering the SUSY version. We see that even at this order the asymptotic freedom for QCD has been lost for energies higher than μ_{331}. The lose of the asymptotic freedom at higher energies, for both the SUSY Model A and B is a prediction of the models since this result does not depend on the value of μ_{331}. Of course, a more careful analysis should be done.

6 Discussions

We have re-examined the question of the non-perturbative limit and the Landau-like pole in 3–3–1 models. In addition we have considered the respective supersymmetric versions and also the situation when the exotic quarks are heavy enough and do not enter in the running equation of α_X. In practice what we have studied is the energy scale at which a model loses its perturbative character, M', or calculated directly the Landau pole, Λ, from (16). We find, as expected, that for all these models these energy scales are of the same order of magnitude, i.e., $M' \lesssim \Lambda$.

From Table 1 we see that for Model A, the values of M' and Λ decrease with the value of μ_{331} but increase for lower μ_{331} if we omit the exotic quarks in the running equation. The maximal values of 4.8 TeV or 8.2 TeV without the exotic quarks, respectively, occur when $\mu_{331} = 500$ GeV. For the respective SUSY cases, we see from Table 2 that M' and Λ always decrease with μ_{331} and also that they have lower values than the respective non-SUSY model. The result for the model with the scalar sextet (Model B) are shown in Tables 3 and 4. The largest value for M' (Λ) is 20.1 (44.2) TeV when the heavy quarks are not considered. As in Model A, both scales also decrease with the value of μ_{331}.

Notice that from Table 1, the value of Λ (or M') for Model A (without SUSY) is always lower than the value obtained in [25–27] and in Fig. 1. As we have mentioned before, the latter value should be an upper limit for Λ. This is confirmed when the extra degrees of freedom of the 3–3–1 model are taken into account, for energies above μ_{331}. As said before, we can see from Tables 1–4, the value

of Λ increases when the scale μ_{331} increases. But, as μ_{331} becomes larger the difference between both energy scales becomes smaller and in some point μ_{331} must be equal to Λ. However, notice also that when we omit from the analyses the exotic quarks this upper bound is evaded. This happens because the right-handed components of those quarks have the largest value of the $U(1)_X$ charge making that α_X run more rapidly compared to the case where only the standard model particles are taken into account. When the heavy quarks are switched off α_X run again slowly and the Landau-like pole occurs at a higher energy. The scenarios without the exotic quarks could be realized if there are strong dynamical effects with these degrees of freedom in this range of energy an probably the number of scalar multiplets of these models may be lower than it has been considered [31]. Notice also that since SUSY implies more degrees of freedom the values of M' and Λ are always lower than in the respective non-SUSY model. If the Landau pole is calculated by using only the degrees of freedom below the 3–3–1 energy and the condition $\sin^2\theta_W(\Lambda) = 0.25$ from (11), the value obtained is shown in Fig. 1. Since the value of μ_{331} is below of these values we have studied how the value of the pole and the perturbative limit are modified when $0.5 \leq \mu_{331} \leq 2$ TeV.

Finally, let us mention that there is another type of 3–3–1 model in which the right-handed neutrinos or heavy neutral leptons belong to the same triplet than the ordinary leptons [32,33]. The charge operator is defined in this case as

$$\mathcal{Q}_B = \frac{1}{2}\left(\lambda_3 - \frac{1}{\sqrt{3}}\lambda_8\right) + X. \quad (25)$$

In this sort of models the Landau pole arise above the Planck scale and for this reason it has no physical consequences. However, models with electric charge operator defined by (2) and (25) are embedded in an $SU(3)_C \otimes SU(4)_L \otimes U(1)_N$ but in this 3–4–1 model the relating the coupling constant g_L and g_X is given also by (1) [34]. Thus, our results are also valid for the case of 3–4–1 models.

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